Compressed Sensing and HYPR

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Introduction

MRI is a powerful medical imaging modality that allows for 2D and 3D imaging in arbitrary scan orientations. However, in some clinical applications its full potential is compromised by the lack of imaging speed, e.g. in dynamic imaging such as interventional MRI or bolus tracking in MR Angiography, or in lengthy acquisitions such as MR spectroscopy, diffusion tensor imaging, 4D phase contrast MRI etc.

Despite major reductions in image acquisition times by advances in gradient and coil hardware, trajectory and sequence design, the use of parallel MRI, and other methods, further decreased acquisition times are a desirable goal. Such accelerated imaging can improve patient comfort, increase patient throughput and reduce motion artifacts. In addition, it can greatly benefit or enable the imaging of dynamic processes including bolus tracking in angiography and function studies, interventional procedures, and motion studies.

Here we discuss methods that purposefully reduce the number of acquired data samples in order to minimize acquisition time. While other approaches that use a priori information on the sampled data have been proposed, such as constrained reconstruction [1] including RIGR [2] and spatio-temporal undersampling with kt BLAST and kt SENSE [3], here we focus on the recently introduced Compressed Sensing [4] and HYPR [5] methods. Both of these approaches are based on the observation that most MR images contain redundant information either in form of compressibility of single images or spatio-temporal correlation among a time series or a functional image series.

Compressed Sensing

The *compressed sensing (CS)* or *compressed sampling* theory was recently established in the information theory community by Candes and Donoho [6, 7]. It states that compressible information can be recovered from significantly less measurements as determined by the Nyquist sampling theorem if the information is compressible and if the measurements are incoherent (e.g. random k-space sampling in MRI). In MRI, less than N^2 Fourier samples can be sufficient to reconstruct an N^2 image if the image is sparse and if the data sampling can occur in a random or very specific deterministic fashion [8].

While some MR images are fairly sparse by nature (e.g. contrast-enhanced MR angiograms with mask subtraction or phase contrast angiograms), other imaging scenes require the incorporation of sparsifying transforms such as the discrete gradient or wavelet transform in the reconstruction process. For example, several approaches use the total variance as the transform basis in MRI [4, 9, 10]. In a somewhat related fashion, digital images stored in the popular jpeg format undergo a discrete cosine transform for sparsifying and can be used with a lossy compression for dramatic reductions in file size with little perceptible loss to the observer. This lecture will discuss the basics of Compressed Sensing including considerations for the choice of the sparsifying transform and k-space sampling pattern.

HYPR

While Compressed Sensing can be applied to a single image, the HYPR (**H**ighl**Y** constrained back**PR**ojection) technique [5] explores spatio-temporal redundancies in an image series.

HYPR achieves high temporal and spatial resolution by severe angular undersampling with an interleaved 2D or 3D stack of stars, or truly 3D radial trajectory. Without further processing, these images would suffer from low SNR and sever streak artifacts. These image degradations are overcome by the incorporation of a reference or 'composite' image reconstructed from the projections in multiple or all time frames. In this technique, the reference data are not acquired before or after the dynamic process of interest but is obtained from the dynamic data instead.

The process is illustrated for the case of contrast enhanced MRA in Fig. 1. Prior to the arrival of injected contrast material, a well sampled mask image is acquired so that signal from static tissue can be suppressed. Then a series of undersampled, interleaved radial data sets are acquired during the contrast pass using a small number of radial projections. As shown in Fig. 1, if these are reconstructed using conventional filtered back projection, severe streak artifacts are seen (top row). For the HYPR algorithm a composite image is formed using all or a portion of the projections acquired during the examination. This composite image must be chosen so that for the number of projections acquired in each time frame, the temporal duration of the time frame projection set rather than the composite duration determines the temporal resolution of the image series. To fulfill this requirement the number of projections must be adequate to diminish signals that might be present in the composite image but which do not belong to the current time frame. Composite images can be formed using all time frames, an increasing sum beginning at the time of contrast arrival (progressive composite) or a sliding window sum centered at the current time frame.

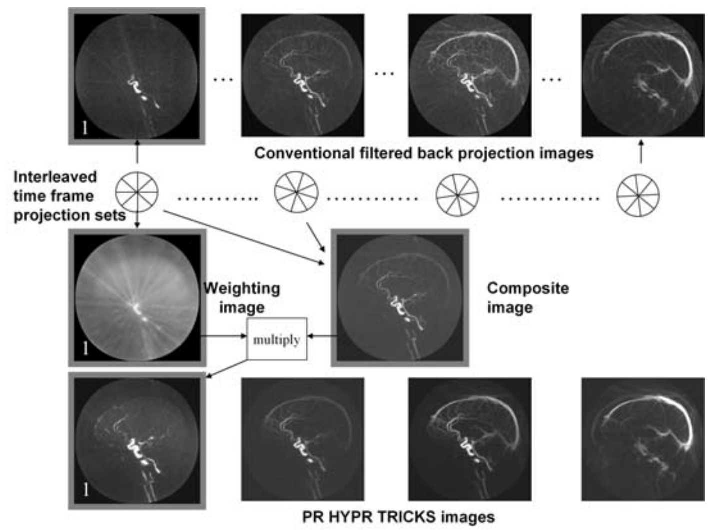

Fig. 1: Schematic of HYPR acquisition and reconstruction.

Each interleaved k-space projection is Fourier transformed to produce image space projections similar to what would be acquired with X-ray CT. Using a set of such projections adequate to satisfy the Nyquist theorem, an exact mathematical reconstruction can be obtained using the standard filtered backprojection algorithm. However as illustrated in Fig. 1, for undersampled data sets the filtered back projection produces severe streaking artifacts. In the HYPR method a weighting image is formed from the sum of projections acquired in each time frame. These projections are divided by the corresponding projection values obtained by Radon transformation of the composite image prior to formation of the weighting image.

As illustrated in Fig. 1, to form each HYPR time frame the weighting image for that frame is multiplied by the composite image. Although the composite image is built from many time frames and contains contrast that does not appear until long after the present time frame, the future signals are suppressed by the weighting image, provided that a sufficient number of projections are used. Typically, angular undersampling factors of about 80 are possible. Fig. 1 shows the improved image quality of the HYPR images (bottom row) relative to the undersampled filtered back projection images (top row). It is also clear that the sagittal sinus that appears in the composite image is suppressed following multiplication by the weighting image for frame 1.

An essential component in HYPR processing is the use of an unfiltered constrained back projection process instead of the standard technique for backprojection when calculating the weighting image. In the filtered back projection case, the detected projection information is filtered and the signal is uniformly spread across the image plane. In the constrained back projection the composite image and backprojected projection information are multiplied resulting in the deposition of signal intensity only in contrast-containing regions. When a sufficient number of projections are used, signals present in the composite but not in the actual time frame are well suppressed so that the temporal resolution is determined by the time frame projections. How well this is accomplished depends on the sparsity of the information in the imaging volume and on the degree of spatio-temporal correlation between various locations in the volume. It is this need to use an adequate number of projections that ultimately determines the acceleration factor that can be used while still maintaining good contrast waveform fidelity.

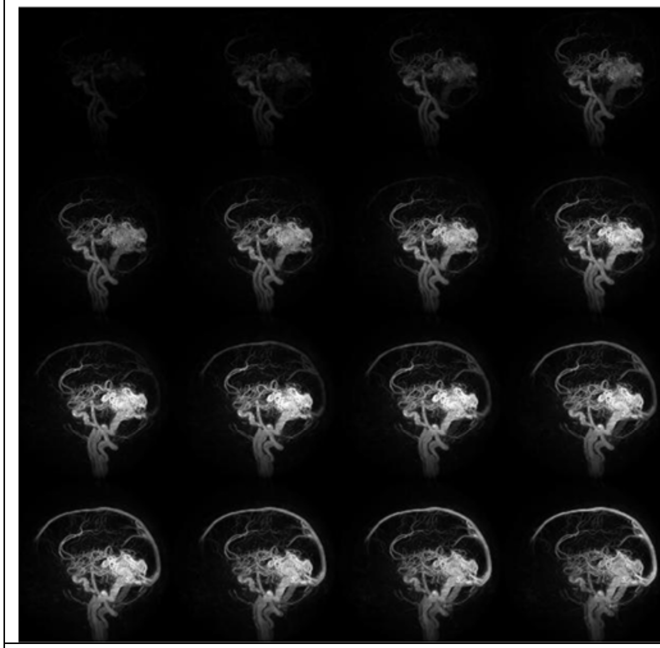

Fig. 2: Contrast-enhanced HYPR VIPR exam with phase contrast reconstruction constraint (PC HYPR FLOW, [11])

More recently, several modifications to the basic HYPR algorithm have been proposed to allow for faster reconstruction via regridding (HYPR LR) [12], the use of a second acquisition with a different contrats mechanism for the generation of the composite image (HYPR FLOW [11]), image reconstruction of phase sensitive data with a complex HYPR algorithm [13], and iterative HYPR reconstruction techniques [14] [15] for improved accuracy for quantitative imaging to permit HYPR processing in a broader range of combinations of sparsity and spatio-temporal correlation. An image series acquired with the PC HYPR Flow technique is shown in Fig. 2. The dataset has a spatial resolution of 320x320x320 voxels with an isotropic spatial resolution of 0.6 mm. Images are reconstructed at 0.75 s time intervals.

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